Assignment I 2MA201, CDE

1. Evaluate $\int_0^{\infty} x^{\frac{1}{4}}\, e^{-\sqrt{x}} dx$.
2. Evaluate $\int_0^{\infty} e^{-n^2x^2}\, dx$.
3. Evaluate $\int_0^1 \frac{dx}{\sqrt{x \log(1/x)}}$.
4. Express $\int_0^{\infty} \left(\frac{x}{1+x^2}\right)^6 dx$ in terms of gamma function and evaluate it.
5. Prove that $\int_0^{\frac{\pi}{4}} \sqrt{\tan 2x}\, dx = \frac{\pi}{2\sqrt{2}}$. (Hint: take $2x = t$)
6. Show that $\int_0^{\infty} \frac{x^2}{(1+x^4)^3} dx = \frac{5\pi\sqrt{2}}{128}$. (Hint: take $x = \sqrt{\tan\theta}$)
7. Find the volume of the right circular cone formed by the revolution of a right angled triangle about a side which contains the right angle.
8. Find the volume of a spherical cap of height h cut off from a sphere of radius a.
9. Find the volume of the solid generated by revolution of the region bounded by the lines $y = 0, 2 + y = 2a$ and the curve $y^2 = \frac{a^2x}{a-x}$ about x-axis.
10. Find the area of the surface of revolution obtained by revolving the graph of the curve $y = x^3$ from $x = 0$ to $x = 1$ about x-axis.
11. Prove that the surface of the solid obtained by revolving the arc of the curve $y = \sin x$ from $x = 0$ to $x = \pi$ about x-axis is $2\pi\,[\sqrt{2} + \log(1 + \sqrt{2})]$.
12. The part of the parabola $y^2 = 4ax$ cut off by the latus rectum revolves about the tangent at the vertex. Find the curved surface area of the reel thus generated.
13. Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$.
14. Evaluate $\iint_R y\, dxdy$, where R is the region bounded by the parabola $y^2 = 4x$ and $x^2 = 4y$.
15. Evaluate $\iint_R r^2 \sin\theta\, drd\theta$, where R is the semi circle $r = 2\,\mathrm{a}\cos\theta$ above the initial line.
16. Evaluate $\int_1^4 \int_{\sqrt{y}}^2 (x^2 + y^2) dA$ by changing the order of integration.
17. Evaluate $\int_0^2 \int_{2-x}^{2+x} x\, dydx$ by changing the order of integration.
18. Change the order of integration and hence evaluate the following double integrals:
    i) $\int_0^1 \int_x^{2-x} \frac{x}{y}\, dydx$.
    ii) $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xy\, dxdy$.
    iii) $\int_0^1 \int_{x^2}^{2-x} xy\, dydx$.
    iv) $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dxdy$

19. Evaluate the following integral by changing into polar co-ordinates

I. $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\, dxdy$

II. $\iint_R \sqrt{4-x^2-y^2}dxdy$, where R is the region bounded by the semi-circle $x^2+y^2-2x=0$, lying in the first quadrant.

III. $\iint \frac{xy}{\sqrt{1-y^2}}dxdy$ over the positive quadrant of the circle $x^2+y^2=1$.

IV. $\iint \frac{x^2y^2}{x^2+y^2}dxdy$ over the annular region between the circle $x^2+y^2=a^2$ and $x^2+y^2=b^2(a<b)$

20. Area by double integration

I. Find the area enclosed within the curve $y=2-x$ and $y^2=2(2-x)$

II. Find the area lying between the parabola $y=4x-x^2$ and the line $y=x$.

III. Find the area common to the circle $r=a$ and $r=2a\cos\theta$.

IV. Find the area enclosed by the curve $y=\sin x$,$y=\cos x$ for $0\le x\le\frac{\pi}{4}$

21. Volume by double integration

I. Find the volume bounded above by the sphere $x^2+y^2+z^2=2a^2$ and below by the paraboloid $az=x^2+y^2$

II. Find the volume common to the cylinder $x^2+y^2=a^2$ and $x^2+z^2=a^2$

III. Find the volume of the solid under the curve $z=\sqrt{x^2+y^2}$ and above $x^2+y^2\le 4$

IV. Find the volume bounded by cylinder $x^2+y^2=4$ and plane $y+z=4$ and $z=0$

22. Triple integration

I. Evaluate $\iiint_R (x^2+y^2+z^2)dxdydz$, where R denotes the region bounded by $x=0$,$y=0$,$z=0$ and $x+y+z=a$,$(a>0)$

II. Evaluate $\iiint_R (x+y+z)dxdydz$ where R:$0\le x\le 1$,$1\le y\le 2$ and $2\le z\le 3$

III. Evaluate $\int_0^1\int_x^1\int_0^{y-x} dzdydx$